Sensor Fusion in Robot Localization using DS-Evidence Theory with Conflict Detection using Mahalanobis Distance

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***Abstract*— This article contains a new approach for combining sensory information for sensors with compatible information. Using Dempster-Shafer Evidence Theory, we show how to combine sensory information by Yager Combination rule. We also present a method to distinguish contrasts between Robots sensors outputs based on Mahalanobis distance, and find the sensor with irrelevant outputs. Experiments on a Robot facilitated with Vision and Encoder shows that this approach reduces localization error significantly, having better performance than Kalman Filter.**

***Index Terms*— Localization, Yager Combination Rule, Dempster-Shafer Evidence Theory, Sensor Fusion, Mobile Robot**

I. INTRODUCTION

ENTERING to an unknown environment for a mobile robots completely depends on the information provided by its sensors to move perfectly and accomplish its mission. It's easy when the gathered information from sensors is consistence and accurate but this case is rare and sensory information is usually inaccurate and inconsistent.

How does the robot deal with uncertainty in sensory information? How does it detect the inconsistency in streams of sensory information? And the main issue belongs to the way it tends to combine these inaccurate information to accomplish the mission. These are questions that this article is to answer. So as a framework, mobile robot localization is chosen.

Sensor-based robot localization has been recognized as one of the fundamental problems in mobile robotics. The localization problem is frequently divided into two sub problems [1],[5]:

- **Position tracking**: which seeks to compensate small dead reckoning errors under the assumption that the initial position is known;
- **Global localization**: which addresses the problem of localization with no a priori information.

According to Cox [2],

> *"Using sensory information to locate the robot in its environment is the most fundamental problem to providing a mobile robot with autonomous capabilities."*

In order to combine non-deterministic sensory information, we use Dempster-Shafer Evidence Theory, one of best tools to combine information gathered from several sources. This theory is used as a substitute of Bayes approach in sensory information combination problem due to better information combination as mentioned in [6] and the ability of evaluating incompatibility in occupancy grid as stated in [7],[13].

Before combining sensory information, the validity of such them must be checked. Mahalanobis distance is employed here to test compatibility of new data with recent ones, before the combination phase take place. As a result only validated information, i.e. those sensory information which are compatible with others, are combined.

In this article, we try to reduce localization error. Information gathered from vision and encoder are combined using Dempster-Shafer Evidence Theory and magnificent reduce in localization error is observed as stated later in the article.

II. LITERATURE REVIEW

Murphy et al. detect inconsistency between gathered information from sensors using a Gambino indicator, and find the sensor responsible of inconsistent information and improved quality of created map significantly [10],[13].

Yi et al. [8] developed a sensing quality metric to adapt their sonar sensor model in an unknown environment. They used Dempster-Shafer to update the occupancy grid and to generate raw conflict evidence. Experimented their solution on Nomadic Super Scout II robot, they gave an example that despite of creating better compatibility, still didn't have enough accuracy regarding real environment map.

Shayer et al. [9] came with a new quality criterion for sensing that benefited of a sensory information combination technique based on Adaptive Fuzzy-Logic. In their approach, a binary occupancy grid is used along with a global network for environment. The agreement rate of combined map and local map of each sensor is considered as relative error, and used to rank the sensors. Using this method, in 83 percent of cases the ranking proved successful.

As shown by recent researches, Dempster-Shafer Evidence Theory has sufficient abilities to combine sensory information for mobile robots as required by Bayes Theory and Fuzzy point of view [5]. In [10] the Dempster-Shafer Evidence Theory is compared by HIMM/VFH network which is presented by [11]. Evidence theory passed all tests at least in the same level of VFH.

In [6] showed the superiority of Evidence Theory to Bayes Theory in updating methods. Also in [12] some Evaluation Structures are featured in which, Evidence Theory includes specification of both Bayes Theory and Fuzzy logic into itself.

III. Approach

In this study, a solution based on Dempster-Shafer Evidence Theory is provided. The solution tends to reduce the error in robot localization. This error is occurred due to incomplete or erroneous sensor information. It's assumed that the robot is only able to gather information through its vision and encoder to find its positioning in the environment.

The drawback of using dead reckoning for localization in this case is obvious, as the summation of error is increasing by time and after a short while it will not be trustable at all to accept it as robot position.

It's assumed that the vision has such defects that due to noise of image or structure of environment, gathered information matches several places of robot's map. In this case, robot may assume itself in another position of environment that is far from the original position, and this continues until newer evidences provided. This problem is usual in some applications such as Robotic Competition that for example a soccer robot assumes an object from out of the field as ball, or can't see the side flags due to positioning of other robots for a while. The goal of this article is to present a solution for combining information of encoder and vision such that localization error reduces to minimum in these cases.

IV. Algorithm

Here, to locate the robot, information gathered from vision and encoder is used. Schematic of process is depicted in Fig. 1. Some definitions are necessary to understand the main work, so they are described as follows.

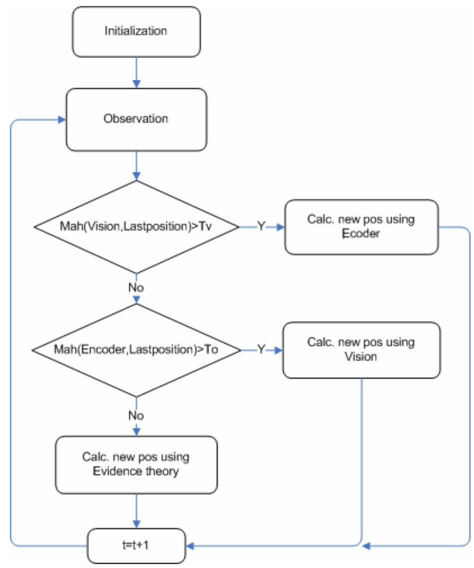

Fig. 1 : Schematic of process

A. Observation

In each step of algorithm, the robot receives the data that sensors measured. It's assumed that the output of encoders provides the distance that reference point (center of robot's main axis) traveled and robot's rotation regarding axis Z in the last step of time that is presented as follows.

$$Z_O^t = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \theta \end{bmatrix} \tag{1}$$

Also it's assumed that output of vision, shows the position of the robot's reference point regarding environment, which is provided after proper calculations:

$$Z_v^t = \begin{bmatrix} x_v \\ y_v \\ \theta_v \end{bmatrix} \tag{2}$$

Because of the position estimated by vision's output in each state does not depend on the previous vision's output estimates, it has no dead reckoning error, thus it can be used to eliminate dead reckoning error of encoder. For this purpose $Z_O^{'}$ can be defined in this way:

$$Z_O^{'} = \sum_{k=l+1 \ldots t-1} Z_O^k \tag{3}$$

Where, l is the number of latest step of time which vision's output was inconsistence.

B. Mahalanobis Distance

Mahalanobis distance [4] is the tool with which it's possible to understand whether the outputs of sensor consistence or not.

The main advantage of this technique is its speed.

$$Mah\left(Z^t, p^{t-1}, \Sigma^t\right) = \left(Z^t - p^{t-1}\right)^T (\Sigma^t)^{-1} \left(Z^t - p^{t-1}\right) \quad (4)$$

Z^t shows the robot position which is calculated using sensor output, and p^{t-1} is its position in the last step of time. Eventually $\sum^t$ is calculated in this way:

$$\Sigma^t = \begin{bmatrix} \left(Z_O^t(1) + Z_O^{'}(1)\right)^2 & 0 & 0 \\ 0 & \left(Z_O^t(2) + Z_O^{'}(2)\right)^2 & 0 \\ 0 & 0 & \left(Z_O^t(3) + Z_O^{'}(3)\right)^2 \end{bmatrix} \quad (5)$$

The maximum mobility of robot is bounded, directly relative to the amount that encoder reported. Note that encoder reports the mobility of the robot more than the real. This is why that $\sum^t$ is defined in such way.

C. *Dempster-Shafer Evidence Theory*

Being consistence, outputs of encoder and vision should be combined next. This task is done by Dempster-Shafer Evidence Theory. Each particle of problem space of the problem our frame of discriminant has two valid states: Yes (the robot is in this position), and No (the robot is not in this position). As Evidence Theory stated, BPA values must be assigned for power set of these criteria.

$$\Omega = \{y, n\} \quad (6)$$

$$P(\Omega) = \{\varnothing, \{y\}, \{n\}, \{y, n\}\} \quad (7)$$

Where, $P(\Omega)$ is the power set of X. We assumed that:

$$m_i(\varnothing) = 0 \quad (8)$$

To calculate the BPA of other elements $P(\Omega)$ of these statements is used:

$$m_i(\{y\}) = \frac{1}{(2\pi)^{n/2}\sqrt{|\Sigma_i|}} \exp\left[-\frac{1}{2}(x - \mu_i)^T \Sigma_i^{-1}(x - \mu_i)\right] \quad (9)$$

In this relation, Σ_i is the covariance of sensor local error and μ_i is the place that sensors output indicates as robot's position. In the case of an encoder, obviously it's the last position of robot moved by output of encoder. In this equation, *T* plays the role of threshold. Where ever the probability of robot presence is lower than this threshold, then it's assumed that robot is not there certainly.

$$m_i(\{n\}) = \begin{cases} 1 - m_i(\{y\}) & m_i(\{y\}) \le T \\ 0 & Otherwise \end{cases} \quad (10)$$

As mentioned in the theory, the summation of BPA values is equal to 1, so:

$$\sum_{A \in P(\Omega)} m(A) = 1 \quad (11)$$

So for $m_i(\{y, n\})$ the equation reforms to:

$$m_i(\{y, n\}) = 1 - m_i(\{y\}) - m_i(\{n\}) \quad (12)$$

This value shows the ignorance of system to robot presence in the given position. Therefore closer values to zero mean more certainty of whether robot is present in this position or not. After calculating the value of m for both sensors and all of positions problem includes, resulted values are combined using Yager Combination Rule. The main advantage of using this rules are indicated in [14]:

1. The rule does not filter or change the evidence through normalization.
2. The allocation of conflict to the universal set (Ω) instead of to the null set ($\varnothing$). Thus mass associated with conflict is interpreted as the degree of ignorance.

The process of combination in this rule is as follows:

$$q(A) = \sum_{B \cap C = A} m_1(B) m_2(C) \quad (13)$$

In the current application of this rule, it reshapes into:

$$q(\{y\}) = m_o(\{y\}) * m_v(\{y\}) + m_o(\{y, n\}) * m_v(\{y\}) + m_o(\{y\}) * m_v(\{y, n\}) \quad (14)$$

$$q(\{n\}) = m_o(\{n\}) * m_v(\{n\}) + m_o(\{y, n\}) * m_v(\{n\}) + m_o(\{n\}) * m_v(\{y, n\}) \quad (15)$$

$$q(\{y, n\}) = m_o(\{y, n\}) * m_v(\{y, n\}) \quad (16)$$

In these equations, m_o and m_v are respectively BPA of encoder and vision. This rule can be generalized through more sensors easily.

At the end the final result of combination must be chosen. To select one of problem space states as the final answer, several different approaches are suggested. In this article, the position with the maximum value of $q(\{y\})$ is selected as the new position of robot in the environment.

D. *The process of the method*

The progress of method is depicted in flowchart 1, with following details:

1. Initialization: Setting initial values of parameters($Z_O^{'}$)

   $$Z_O^{'} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T, \; t = 1, \;\; p^0 = init\ pose \quad (17)$$

2. Observation: Sensor output is observed as described in section IV.A
3. Mahalanobis distance of vision output (Z_V^t) and last robot position (p^{t-1}) is calculated using the equation (4). If it exceeds the T_v threshold, means that vision output has some conflicts. Thus encoder output is accepted, so new position of robot is calculated in the

following way. In addition, the value of $Z_O^{'}$ is updated too.

$$p^t = p^{t-1} + Z_O^t \quad (18)$$

$$Z_O^{'} = Z_O^{'} + Z_O^t \quad (19)$$

The T_v has been determined using vision properties, such as variance of vision errors.

4. If the vision doesn't have any conflicts, the Mahalanobis distance of encoder ($p^{t-1} + Z_O^t$) with last position of robot is calculated based on equation (4). If the distance exceeds T_O, the output of encoder is ignored and output of vision will be used.

$$p^t = Z_V^t \quad (20)$$

$$Z_O^{'} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \quad (21)$$

The T_O has been determined using encoder properties, such as variance of encoder errors.

5. If none of sensors conflicts, their outputs are combined using Evidence Theory. This task is performed as described in section IV.C. Also:

$$Z_O^{'} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \quad (22)$$

6. Let $t = t + 1$ and then return to step 2.

V. Simulation Results

All simulations of this article were made by Matlab software, using SimRobot toolbox. Having added vision and encoder sensor to the simulator, the robot is commanded to surf on a specific path. During the simulation, outputs of robot sensors and the real position of robot (called Ground Truth) are collected and saved. This provided a dataset for further experiments and validation of results.

Followed are results of algorithm performed on two datasets, D1 and D2, and related Kalman filter results on the same datasets. Kalman Filter is implemented as described in [3].

The blue path represents ground truth, the green path represents the path estimate from vision alone (Fig. 2 and Fig. 6), the black path represents the path estimate from dead reckoning alone(Fig. 3 and Fig. 7) and the red path represents the path estimate by the method devised in this article (Fig. 5 and Fig. 9) or Kalman filter (Fig. 4 and Fig. 8).

As it's visible in Fig. 2 and Fig. 6, the output of robot is not trustable, because the vision gives an unknown position of map as the current position of it. This is why Kalman filter has poor performance on this dataset; this filter can't distinguish the contrast between input information and is not able to deal with this unawareness. As a matter of fact Kalman filter gradually get closer to the virtual point that vision spotted as robot current position.

Fig. 5 and Fig. 9 show that this algorithm had good performance on this case. In addition to depicted evidences of this superiority, Table I prove this, using sum square error of these methods for mentioned datasets.

At the end it should be mentioned that our method, such as most other methods which use Evidence Theory, has complexity problem. Therefore, when this complexity makes the method inefficient, using topological map representations will be helpful.

A. Dataset D1

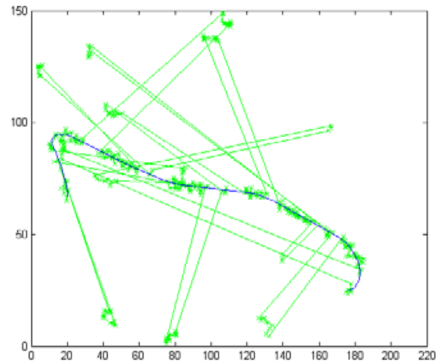

Fig. 2 : The path estimated from vision

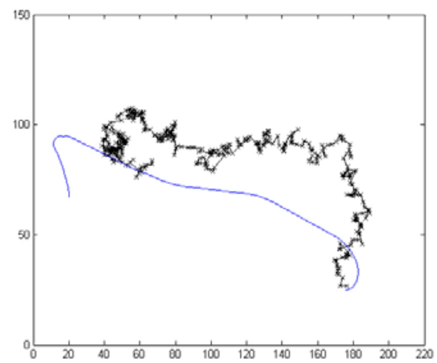

Fig. 3 : Path estimated from encoders

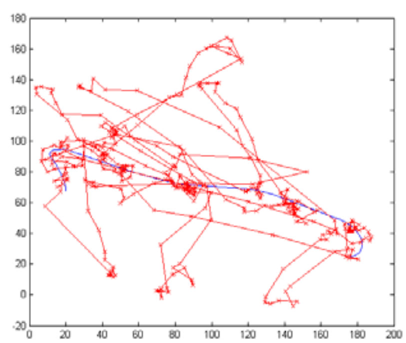

Fig. 4: Path Estimated from Kalman Filter localization

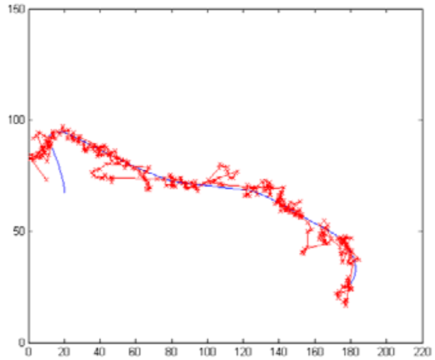

Fig. 5: Path Estimated from Our Method

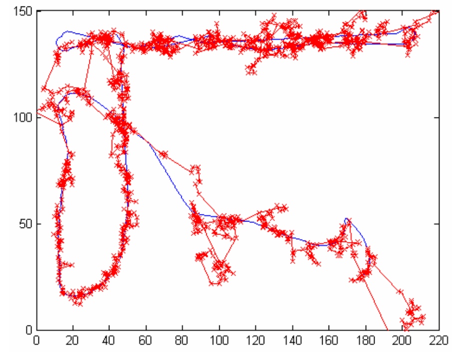

Fig. 9: Path Estimated from Our Method

Table I

SUM OF SQUARE ERRORS FOR DIFFERENT LOCALIZATIONS

Localization method	Encoder	Vision	Kalman	Our Method
Dataset D1 ($*10^5$)	7.7781	2.9595	7.1227	0.3377
Dataset D2 ($*10^6$)	5.7422	4.2788	5.0283	0.1546

B. *Dataset D2*

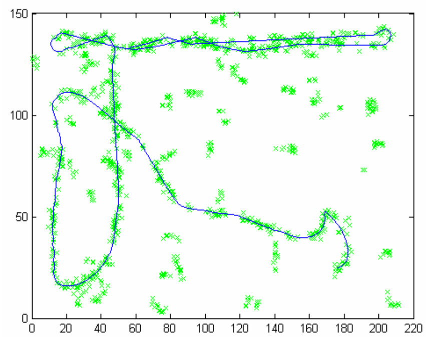

Fig. 6 : The path estimated from vision

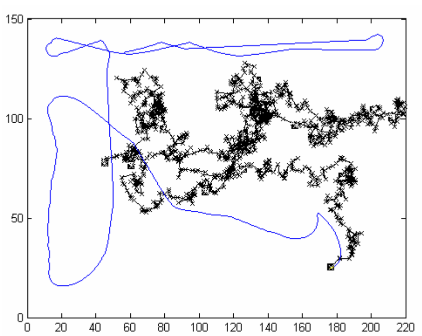

Fig. 7: Path estimated from encoders

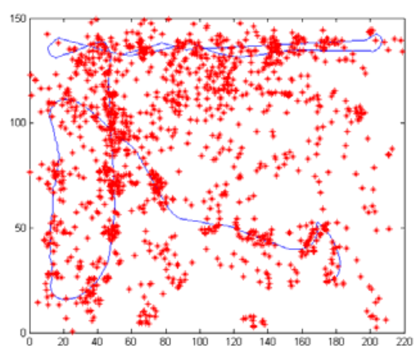

Fig. 8: Path Estimated from Kalman Filter localization

VI. CONCLUSION

This article contains a new approach to combine sensory information using Dempster-Shafer Evidence Theory and Mahalanobis Distance. With assist of this method, it's possible to detect sensors which suffer from temporal malfunction and avoid their bad effect on total result. In general, this method is able to find the sensor which has inconsistence information with other sensors and with prior knowledge of the environment.

To combine sensory information, this article take advantage of Yager Combination Rule, which relates the conflicts to reference set instead of $\varnothing$. Thus, conflicts are treated as ignorance of the agent from the environment and have less effect in combination phase output.

Applied on two datasets, the featured method showed that it is capable of managing cases in which sensory information have immediate changes and outperformed Kalman filter in such cases, while this filter is not able to discriminate inconsistency between data, nor understands the ignorance implied in presented information.

REFERENCES

[1] Dieter Fox, Wolfram Burgard, Hannes Kruppa, Sebastian Thrun. “A Probabilistic Approach to Collaborative Multi-Robot Localization”. Auton. Robots 8(3): (2000), pp. 325-344

[2] I.J. Cox and G.T.Wilfong, editors. *Autonomous Robot Vehicles*. Springer Verlag, 1990.

[3] Roland Siegwart and Illah R. Nourbakhsh. *Introduction to Autonomous Mobile Robots*. The MIT Press. 2004

[4] Chulhee Lee and David Landgrebe, ”Feature Extraction and Classification Algorithm For High Dimensional Data”, PhD Thesis and School of Electrical Engineering Technical Report TR-EE 93-1, January, 1993 .

[5] Murphy, R.R., “Dempster-Shafer Theory for Sensor Fusion in Autonomous Mobile Robots”, IEEE Transactions on Robotics and Automation, vol. 14, no.2, April, 1998, pp. 197–206.

[6] Pagac, D.; Nebot, E.; and Durrant-Whyte, H. “An evidential approach to map-building for autonomous vehicles”. IEEE Transactions on Robotics and Automation 14(4): (1998) pp. 623–629.

[7] Carlson, J.; Murphy, R.; Christopher, S.; and Casper, J. 2005. “Conflict metric as a measure of sensing quality”. In Proceedings of the IEEE International Conference on Robotics and Automation (ICRA), to appear.

[8] Yi, Z.; Khing, H. Y.; Seng, C. C.; and Wei, Z. X. “Multi-ultrasonic sensor fusion for mobile robots”. In Proceedings of the IEEE Intelligent Vehicles Symposium (IV), (2000), pp. 387–391.

[9] Shayer, G.; Cohen, O.; Edan, Y.; and Korach, E. “An adaptive fuzzy-logic procedure for ranking logical sensory performance”. In Proceedings IEEE Sensors, volume 2, (2002). pp. 1317–1322.

[10] Murphy, R.; Gomes, K.; and Hershberger, D. “Ultrasonic data fusion as a function of robot velocity”. In Proceedings of SPIE Sensor Fusion and Distributed Robotic Agents, (1996). pp. 114–126.

[11] Borenstein, J., and Koren, Y. “Histogrammic in-motion mapping for mobile robot obstacle avoidance”. IEEE Transactions on Robotics and Automation 7(4). (1991). pp. 535–539.

[12] Zhu, H., and Basir, O. “A scheme for constructing evidence structures in dempster-shafer evidence theory for data fusion”. In Proceedings IEEE International Symposium on Computational Intelligence in Robotics and Automation, volume 2, (2003). pp. 960–965.

[13] Jennifer Carlson, Robin R. Murphy: “Use of Dempster-Shafer Conflict Metric to Adapt Sensor Allocation to Unknown Environments”. FLAIRS Conference. (2006). pp. 866-867

[14] K. Sentz and S. Ferson. “Combination of Evidence in Dempster-Shafer Theory”. Sandia Report. Sandia National Labratories. April 2002.